XXIX. *Theory of the Parallaxes of Altitude for the Sphere, by Mr.* F. Mallet, *Professor and Astronomer at* Upsal; *Translated from the* French *by* M. Maty, *M. D. R. S. Sec.*

Upsal, October 25, 1766.

Read Nov. 20, 1766.

§ 1. LET P be = the moon's horizontal parallax, or 1 to sin. P, as the moon's distance to the radius of the terrestrial sphere, on which the spectator is supposed to be placed. Let A be the distance of the moon from the zenith, and p the parallax of altitude for the same distance. The astronomers usually compute the value of p in the following manner: let sin. p=sin. P. Sin. A, and p' being found by the tables of logarithmic sines, sin. p''= sin. P. sin. $\overline{A+p'}$ is found in like manner, p'' being assumed for the true parallax, which is not accurate.

§ 2. In order to shew this, I have given another method of computing the parallax of altitude, as exactly as may be, by means of the common tables, in the following manner. Since sin. p = sin. P sin. $\overline{A+p}$, we have sin. p = sin. P sin. A cos. p + sin. P cos. A × sin. p, or sin. p (1 − sin. P cos. A) = sin. P sin. A cos. p; hence tang. $p = \frac{\text{sin. P sin. A}}{1-\text{sin. P cos. A}}$. This formula seems a little difficult to be wrought in numbers, but it is as easy as the above one; for, supposing sin. B^2= sin.

fin. P cof. A, the tables will give the angle B, and tang. $p = \frac{\text{fin. P fin. A}}{\text{cof. B}^2}$, the computation of which can give no trouble. Hence it appears, that the calculus for finding the true parallax is not more difficult than that, which gives the faid parallax with an error, the value of which is unknown; for it is evident that the above computation for finding p'' is only an approximation, and that, to make it accurate, it would be neceffary to carry it ftill on by finding fin. $p''' =$ fin. P fin. $(\overline{A + p''})$, and afterwards fin. $p'''' =$ P fin. $(\overline{A + p''})$ &c.

§ 3. I therefore think myfelf in the right to prefer my method to that hitherto ufed by aftronomers. To confirm my opinion, I made a trial, by putting $P = 59'$ and $A = 30°$, and found $p - p'' = 0'',43$, in which the error of the ufual computation amounts to near half a fecond; I therefore give the preference to the geometrical calculus.

§ 4. Before I quit the formula tang. $p = \frac{\text{fin. P fin. A}}{1 - \text{fin. P cof. A}}$, I muft obferve, that the computation of p may be executed by other methods to the fame exactnefs. If we take cof. 2 C = fin. P cof. A, we fhall have tang. $p = \frac{\text{fin. P fin. A}}{2\ (\text{fin. C})\ 2}$, and the computation of this new formula is extremely eafy.

§ 5. The formula tang. $p = \frac{\text{fin. P fin. A}}{1 - \text{fin. P cof. A}}$, gives befides, fin. $p = \frac{\text{fin. P fin. A}}{\sqrt{1 + \text{fin. P}^2 - 2\ \text{fin. P cof. A}}}$; make fin. P = 2 cof. D, D being a given angle, of which we may have tables ready made, and we fhall have fin.

$p =$

$$p = \frac{\text{ſin. P ſin. A}}{\sqrt{1 + 2\,\text{ſin. P}\,(\text{coſ. D} - \text{coſ. A})}} = \frac{\text{ſin. P ſin. A}}{\sqrt{1 + 4\,\text{ſin. P ſin.}\frac{A+D}{2}\,\text{ſin.}\frac{A-D}{2}}}$$

; ſince coſ. D–coſ. A $= 2\,\text{ſin.}\frac{A+D}{2}\,\text{ſin.}\frac{A-D}{2}$. This being found without any logarithmic computation, we ſhall find tang. $E^2 = 4\,\text{ſin. P ſin.}\frac{A+D}{2}\,\text{ſin.}\frac{A-D}{2}$, if $A > D$, and hence we may eaſily compute ſin. $p =$ ſin. P ſin. A coſ. E; but if $A < D$ we ſhall find coſ. $F^2 = 4\,\text{ſin. P ſin.}\frac{A+D}{2}\,\text{ſin.}\frac{A-D}{2}$ and hence ſin. $p = \frac{\text{ſin. P ſin. A}}{\text{ſin. F}}$.

§ 6. Similar formulæ may be found for coſ. p, but as the angle p is pretty ſmall, one might eaſily fall into ſome error by the uſual tables of logarithms. I ſhall not ſay what would be the amount of this error of p, having furniſhed the manner of avoiding it; but this remark has not, I think, as yet been made in aſtronomical calculations; and I have found it of great conſequence in computing eclipſes, where the diſtances to be found are very ſmall arches.

§ 7. It may moreover be obſerved, that if $A = D$, ſin. $p =$ ſin. P ſin. A; hence $p' = p$ in the ſame caſe, and $p'' > p$, which ſeems very odd; but the moon then is below the ſenſible horizon.

Theory of the apparent Diameters of the Moon.

§ 1. Firſt the expreſſion of horizontal diameter of the moon, or of the diameter ſeen at the horizon, ſeems to me too vague; for one ought to underſtand by it the diameter ſeen at the center of the

terreſtrial

terrestrial sphere, rather than the apparent diameter at the horizon, which is not affected by refraction. Without this, if the one was confounded with the other, an error would arise for the latitude of Paris from $0'',25$ to $0'',32$.

§ 2. Let us keep the same denominations of P, p, and A, and call D the apparent semi-diameter of the moon at the centre of the sphere, and $d=$ the apparent semi-diameter of the moon at the zenith distance $=$A. We shall have $\text{sin. } A : \text{sin. } \overline{A+p} :: \text{tang. } D : \text{tang. } d$, or if one will, $\text{sin. } A : \text{sin. } \overline{A+p} :: D : d$: the error not exceeding an 100th part of a second.

§ 3. We had above $\text{sin. } p = \text{sin. } P \text{ sin. } \overline{A+p}$. Hence $\text{sin. } P \text{ sin. } A : \text{sin. } p :: (\text{tang. } D : \text{tang. } d) :: D : d$, or because $\text{sin. } p = \frac{\text{cos. } p \text{ sin. } P \text{ sin. } A}{1 - \text{sin. } P \text{ cos. } A}$, $1 - \text{sin. } P \text{ cos. } A : \text{cos. } p :: D, d$, and $d = \frac{D. \text{cos. } p}{1 - \text{sin. } P \text{ cos. } A}$.

§ 4. Mr. Euler, in the Memoirs of the Academy of Berlin, 1747, pag. 175, makes this same value $= \frac{V}{1 - p^2 \text{ sin. } b}$, and according to him, $V = D. M = \text{sin. } P$ $\text{sin. } b = \text{cos. } \overline{A+p}$; from whence it appears, that the true value of the apparent diameter of the moon, is not more difficult to be computed than the approximated one of Mr. Euler, the exact and geometrical formula being $\text{tang. } d = \frac{\text{tang. } D \text{ cos. } p}{1 - \text{sin. } P \text{ cos. } A}$ and that of Mr. Euler $d = \frac{D}{1 - \text{sin. } P \text{ cos. } \overline{A+p}}$; for in both, the values of D, A and p must be employed.

§ 5.

§ 5. It likewiſe appears to me, that ſince $\frac{\text{coſ. } p}{1 - \text{ſin. P coſ. A}} = \frac{\text{ſin. } p}{\text{ſin. P ſin. A}}$ and therefore tang. $d = \frac{\text{tang. D ſin. } p}{\text{ſin. P ſin. A}}$, aſtronomers ought no leſs to employ this laſt formula, than any other more troubleſome, in practical computation. The ſimpleſt is tang. $d = \frac{\text{tang. D ſin. } \overline{\text{A} + p}}{\text{ſin. A}}$, upon the ſuppoſition of an exact table of the parallaxes of altitudes ready made; and I believe it will be as eaſy to compute with tangents as with arches, by means of logarithms; and therefore this ſimplification in putting arches inſtead of tangents is unneceſſary.

§ 6. To try the conſequences of this theory, I made $A = 30°$, $D = 15'$, and taking the vertical of Upſal to the terreſtrial axis for the radius of the ſphere, I found $P = 55', 10'', 3$, ſuppoſing that the axis of the earth, is to the diameter of the equator as 199 to 200, and by the formulæ tang. $d = \frac{\text{tang. D ſin. } \overline{\text{A} + p}}{\text{ſin. A}} = \frac{\text{tang. D coſ. } p}{1 - \text{ſin. P coſ. A}} = \frac{\text{tang. D ſin. } p}{\text{ſin. P ſin. A}}$, I found $d = 15', 12''.664$, but by the formula $d = \frac{\text{D coſ. } p}{1 - \text{ſin. P coſ. A}}$, I had $d = 15', 12'', 675$. and laſtly by that of Euler $d = \frac{\text{D}}{1 - \text{ſin. P coſ. } \overline{\text{A} + p}}$ we have $d = 15', 12'', 635$; from whence it appears that the error is very ſmall, but that with the ſame trouble one may avoid any error whatſoever.

§ 7. The preſent caſe did not give an error of $0'', 001$ in ſubſtituting 1 or the radius inſtead of coſ. p. Hence I conclude that $d = \frac{\text{D}}{1 - \text{ſin. P coſ. A}}$ will be

a more

a more exact formula than that of Euler $d = \frac{D}{1 - \text{ſin. P coſ. } \overline{A+p}}$.

§ 8. By taking $d = \frac{D}{1 - \text{ſin. P coſ. A}}$, we have $d - D = \frac{D \text{ ſin. P coſ. A}}{1 - \text{ſin. P coſ. A}} = \frac{D \text{ ſin. } p \text{ coſ. A}}{\text{ſin. A coſ. } p} = \frac{D \text{ tang. } p}{\text{tang. A}}$, which affords an elegant theorem, to find the increaſe of the apparent diameter of the moon.

§ 9. I have found others by the following methods. Since ſin. A : ſin. $\overline{A+p}$ :: tang. D : tang. d, and ſin. A : ſin. $\overline{A+p}$ − ſin. A :: tang. D : tang. d − tang. D :: ſin. D coſ. d : ſin. $\overline{d-D}$; but coſ. D = coſ. d without any ſenſible error, and ſin. D coſ. D = $\frac{1}{2}$ ſin. 2 D, and ſin. $\overline{A+p}$ − ſin. A = 2 ſin. $\frac{1}{2}$ p coſ. $\overline{A+\frac{1}{2}p}$, we ſhall have ſin. $\overline{d-D} = \frac{\text{ſin. 2 D ſin. } \frac{1}{2}p \text{ coſ. } \overline{A+\frac{1}{2}p}}{\text{ſin. A}}$. In the ſame manner, as I before found ſin. $p' =$ ſin. P ſin. A and ſin. P ſin. A : ſin. p :: tang. D : tang. d, hence ſin. p' : ſin. p − ſin. p' :: ſin. p' : 2 ſin. $\frac{\overline{p-p'}}{2}$ coſ. $\frac{\overline{p+p'}}{2}$ :: $\frac{1}{2}$ ſin. 2 D : ſin. $\overline{d-D} = \frac{\text{ſin. 2 D ſin. } \frac{\overline{p-p'}}{2} \text{ coſ. } \frac{\overline{p+p'}}{2}}{\text{ſin. P ſin. A}}$.

§ 10. Laſtly let L = the diſtance of the moon from the center of the ſphere, l its radius, that of the ſphere being = 1, we have 1 : L :: ſin. P : 1 and L : l :: 1 : tang. D or 1 : l :: ſin. P : tang. D = l ſin. P; hence $l = \frac{\text{tang. D}}{\text{ſin. P}}$ being once found, ſince ſin. A : ſin. $\overline{A+p}$ :: tang. D : tang. d, and ſin. $\overline{A+p}$: ſin. p :: 1 : ſin. P, we ſhall have ſin. A : ſin. p :: tang. D : ſin. P tang. d :: l : tang. $d = \frac{l \text{ ſin. } p}{\text{ſin. A}}$. I found the logarithm of l =

9.4343965 at Upſal, by putting 10 for that of the radius of the ſphere determined as before.

F. Mallet.

XXX. *A Catalogue of the* Fifty Plants *from* Chelſea Garden, *preſented to the* Royal Society *by the worſhipful Company of Apothecaries, for the Year* 1765, *purſuant to the Direction of Sir* Hans Sloane, *Bart. Med. Reg. et Soc. Reg. nuper Præſes: By* William Hudſon, *Societatis Regiæ & clariſſ. Societatis Pharmaceut.* Lond. *Soc. Hort.* Chelſean. *Præfectus et Prælector Botanic.*

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2151 ACHYRANTHES lanata, caule proſtrato, ſpicis ovatis lateralibus, calycibus tomentoſis. Lin. Sp. pl. 296. Mill. Dict. tab. 11. fig. 1.

Amaranthus Indicus verticillatus albus, foliis lanugine incanis. Pluk. alm. 27. tab. 79. f. 8.

2152 Andrachne procumbens herbacea. Lin. Sp. pl. 1439.

Telephoides Græcum humifuſum flore albo. Tourn. cor. 50. Dill. Hort. Elth. 377. tab. 282 f. 364.

2153 Bryonia Africana, foliis palmatis quinquepartitis utrinque lævibus: laciniis pinnatifidis. Lin. Sp. pl. 1438.